Ring Circulator Theory, Design, and Performance

SAMUEL D. EWING, JR., MEMBER, IEEE, AND JERALD A. WEISS, SENIOR MEMBER, IEEE

Abstract—A compact, symmetrical, three-port S-band circulator composed of reciprocal T junctions and nonreciprocal phase shifters is investigated theoretically, and its experimental performance results are presented. The comparison of these results demonstrates that 1) circulators can be designed and their experimental performance described from a network model and 2) there is no theoretical limitation on the minimum amount of total differential phase shift necessary for perfect circulation. Bandwidth is investigated and techniques are discussed, including the introduction of a "backward wave" phase shifter, for achieving larger bandwidths. The stripline nonreciprocal comb-filter phase shifter used in the ring circulator is described and performance results are given.

I. Introduction

Several papers have appeared in the literature proposing the idea of synthesizing a ring circulator.[1]–[3] In particular, Weiss[4] recently performed a rigorous theoretical analysis of the ring concept from a network model (hereinafter referred to as RNM) and showed that perfect circulation could exist in a three-port ring consisting of specially mismatched T junctions connected by phase shifters exhibiting small values of nonreciprocal phase shift.

Previous proposals on ring circulators incorporated relatively large amounts of nonreciprocal phase shift. The embodiments suggested by Kock,[3] Vartanian,[2] and Grace and Arams[1] specify a required total nonreciprocal phase shift of 180 degrees; the last authors interpret a theorem of Carlin's[5] as implying that no physically realizable three-port circulator can have less than 180 degrees of nonreciprocity. The present theory predicts that no such minimum exists. A clarification of this matter will be made later in Section VII.

After a brief review of the RNM, we give a summary of the computational design procedure, followed by experimental results on an S-band stripline version of the ring circulator which was used to test the theory. The use of a filter as a means for producing the required nonreciprocal phase shift is presented. We discuss the relation between bandwidth of the ring circulator and the dispersive characteristics of its components.

II. The Ring Circulator Network Model

A stripline ring circulator is shown in Fig. 1. The ring network representation of the circulator shown in Fig. 2 consists of three symmetrical reciprocal T junctions (T) and three interconnecting nonreciprocal ferrite phase shifters (PS). The analysis involves the calculation of a set of parameters characterizing the phase shifters and the T junctions such that the overall scattering coefficients of the network have values appropriate for perfect circulation. The parameters for the T junctions are the components r, s, r_d, and s_d of the scattering matrix of a physically realizable symmetrical T (see Fig. 3), and for the phase shifters they are the average insertion phase factor $\epsilon=\exp\, i(\theta_+ + \theta_-)/2$ and the nonreciprocal (differential) phase factor $\delta=\exp\, i(\theta_+ - \theta_-)/2$, where θ_+ and θ_- refer to clockwise and counterclockwise phase shift[1] between ports of the circulator, respectively.

Internal scattering within the ring was characterized in terms of waves denoted by C and D, which were composed of contributions due to the transmission and reflection properties of the T junctions. As an example (see Fig. 4), C_{12} is the wave composed of contributions due to the transmission of the unit signal into the sector 1-2, the transmission of the wave C_{31} from sector 3-1 into sector 1-2, and the reflection of D_{21} at T junction 1

$$C_{12} = s_d + sC_{31}e^{-i\theta_+} + rD_{21}e^{-i\theta_-}.$$

A set of six equations can be written in matrix form describing the internal waves. The determinant of the matrix of coefficients is given by

$$\Delta = (R^2 - S^2)^3 - 3R^2(R^2 - S^2 - 1) + (\delta^3 + \delta^{*3})S^3 - 1 \tag{1}$$

where

$$\epsilon = e^{-i(\theta_+ + \theta_-)/2} \tag{2a}$$

$$\delta = e^{-i(\theta_+ - \theta_-)/2} \tag{2b}$$

$$R = r\epsilon \tag{2c}$$

$$S = s\epsilon. \tag{2d}$$

Once the Cs and Ds are known, the expressions for the scattered waves E_1, E_2, and E_3 can be written. For example, consider E_3

$$E_3 = -\delta^*\epsilon \frac{s_d^2}{\Delta}\{(R-S)^3(R+S) - 2R(R-S) + \delta^3 S[1-(R-S)^2] + 1\}. \tag{3}$$

When the condition $E_3=0$ for perfect isolation is imposed, there results either $|E_2|=1$ for perfect circulation or $|E_1|=1$ for complete reflection at the input. The latter result is discarded as trivial because no energy is coupled into the ring.

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S. D. Ewing, Jr., is with Lincoln Lab., Massachusetts Institute of Technology, Lexington, Mass.
J. A. Weiss is with the Dept. of Physics, Worcester Polytechnic Institute, Worcester, Mass. He is also a Consultant to Lincoln Lab., Massachusetts Institute of Technology.

[1] For an explanation of the phase sign convention used in this work, see page 626.[3]

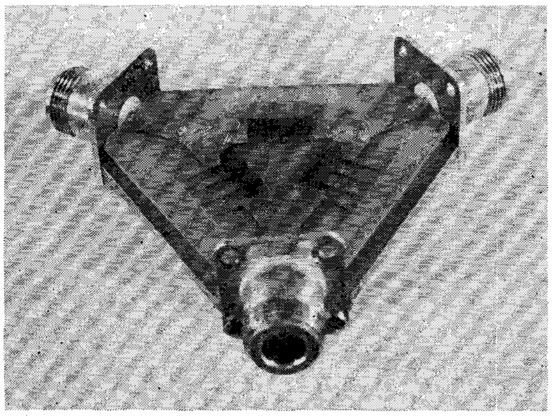

Fig. 1. Stripline S-band ring circulator, shown with magnet and upper ground plane removed.

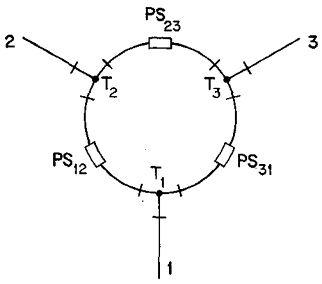

Fig. 2. Schematic diagram of the ring network for a three-port circulator. T denotes reciprocal T junctions, and PS denotes nonreciprocal phase shifters.

Substituting all of (2) into (3) and setting $E_3=0$ results in an expression for δ in terms of ϵ

$$\delta^3 = \frac{a_4\epsilon^4 + a_2\epsilon^2 + a_0}{a_3\epsilon^3 + a_1\epsilon}\cdot \tag{4}$$

With the requirement that $|\delta| = |\epsilon| = 1$, (4) yields a biquartic equation for ϵ

$$A_8\epsilon^8 + A_6\epsilon^6 + A_4\epsilon^4 + A_2\epsilon^2 + A_0 = 0 \tag{5}$$

where the coefficients[4] $A_0, \cdots, A_8$ and $a_0, \cdots, a_4$ involve only the scattering coefficients r and s of the T junctions, thereby making it possible to obtain the δ and ϵ necessary for perfect circulation when incorporating a particular T junction.

The RNM in its original form does not lend itself directly to a synthesis procedure, but concentrates instead on the problem of determining the range of physically realizable T junctions for which perfect circulation is possible. The scattering coefficients of T junctions satisfying reciprocity, energy conservation, and T symmetry were obtained by the standard method of the theory of group representations.[6] The approach yielded a scattering matrix S_T (see Fig. 3) expressed in terms of the complex eigenvalues, s_a, s_b, s_c, and the degeneracy parameter γ. The scattering coefficients are given by

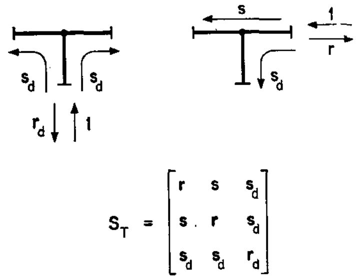

Fig. 3. Definition of the scattering matrix of a symmetrical T junction.

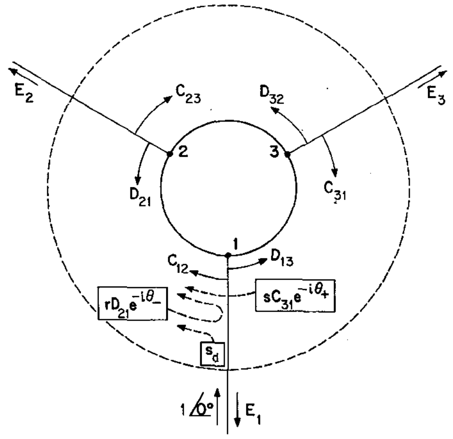

Fig. 4. Internal and scattered waves in the ring network.

$$r = \frac{1}{2}(s_a + s_b\cos^2\gamma + s_c\sin^2\gamma) \tag{6}$$

$$s = \frac{1}{2}(-s_a + s_b\cos^2\gamma + s_c\sin^2\gamma) \tag{7}$$

$$r_d = s_b\sin^2\gamma + s_c\cos^2\gamma \tag{8}$$

$$s_d = \frac{1}{\sqrt{2}}(s_b - s_c)\cos\gamma\sin\gamma \tag{9}$$

in which $|s_a| = |s_b| = |s_c| = 1$ and γ is real.

III. Design

A computational procedure was designed, based on the RNM, whereby circulator parameters can be determined by either of two approaches. Either the parameters δ and ϵ of the phase shifters can be assumed and the corresponding T-junction scattering coefficients determined, or vice versa. We used this latter method, namely assuming the properties of the Ts and deducing the phase-shifter requirements for perfect circulation. This procedure has the advantage that a single style of T-junction design, incorporating a small number of variable tuning elements, may be investigated in a continuous way. Our experience with experimental phase-shifter designs indicated that they have sufficient flexibility in adjustment of ϵ and δ to meet the requirements specified by the computation.

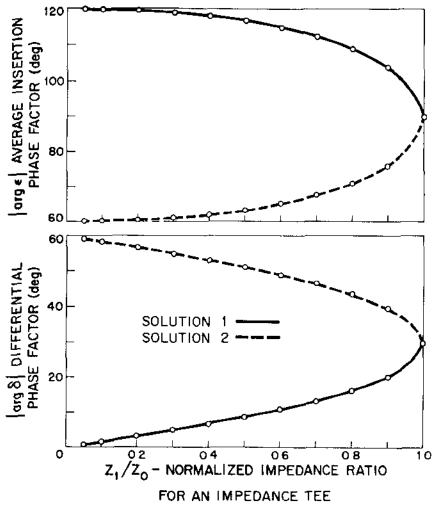

Fig. 5. Solutions for an impedance T. Arguments of the insertion phase factor ϵ and differential phase factor δ required for circulation.

As an illustration of the procedure, consider a stripline T in which the characteristic impedance Z_1 of the symmetrical arms is different from that, Z_0, of the input arm. A computer program was written to calculate the required values of δ of ϵ from (4) and (5) using the scattering coefficients of this simple impedance T junction

$$r_d = \frac{B-2}{B+2},$$

$$s_d = \frac{\sqrt{2B}}{B+2},$$

$$r = \frac{-B}{B+2},$$

$$s = \frac{2}{B+2} \tag{10}$$

where $B = Z_1/Z_0$. The computation resulted in four solutions $(\pm\delta_1, \pm\epsilon_1)$, $(\pm\delta_2, \pm\epsilon_2)$ as functions of B (see Fig. 5) which satisfy all requirements for physical realizability. The behavior of $|\arg \delta_1|$ shows a reduction in the required nonreciprocal phase shift for decreasing values of B.

Other simple T junctions were assumed in which the scattering was controlled by a reactive element located at or near the junction. The objective was to find a T junction which would be easy to fabricate and could be easily adjusted. Fig. 6(a) and (b) are graphs of arg δ and arg ϵ as functions of the reactance magnitude. (Discussion of these curves will be postponed to Section VI.)

The simple symmetrical stripline T of Fig. 6 incorporating a shunt capacitance at the junction was selected for the experimental verification of the theory. A shunt reactance magnitude of 50 ohm which specifies arg $\epsilon = 135$ degrees and arg $\delta = 15$ degrees (nonreciprocal phase shift per sector of 2 arg $\delta = 30$ degrees) was selected for the experimental circulator.

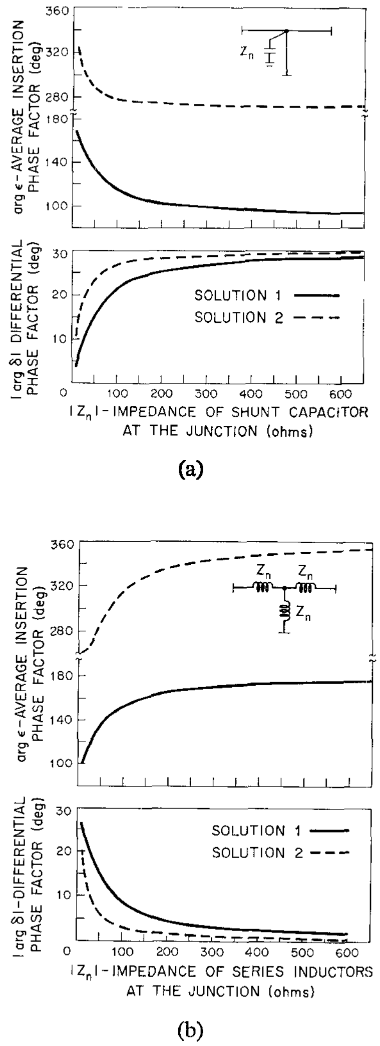

Fig. 6. (a) Solutions for a T with shunt capacitance. (b) Solutions for a T with series inductance.

IV. Experimental Performance

A complete ring circulator was constructed consisting of capacitive shunt screw tuners located over the center of each T junction and three stripline phase shifters biased by an external electromagnet. The experimental performance is shown in Fig. 7. The isolation was in excess of 20 dB from 3.015 to 3.075 GHz (maximum of greater than 40 dB at 3.045 GHz) and the insertion loss was less than 1 dB from 2.950 to 3.165 GHz (with a minimum of 0.4 dB). The VSWR remained less than 1.25 from 3.011 to 3.084 GHz. A comparison of experimental and theoretical parameters is given in Table I.

The close agreement between theory and experiment is evidence that the network model is a sound basis for the circulator design. However, the one exception is in the 20 dB bandwidth, where theory and experiment give 87 and 60 MHz, respectively. This difference may be due to slight deviations from three-fold symmetry, but in any case it deserves further investigation. Agreement between theory and experiment encompasses more detailed data not included

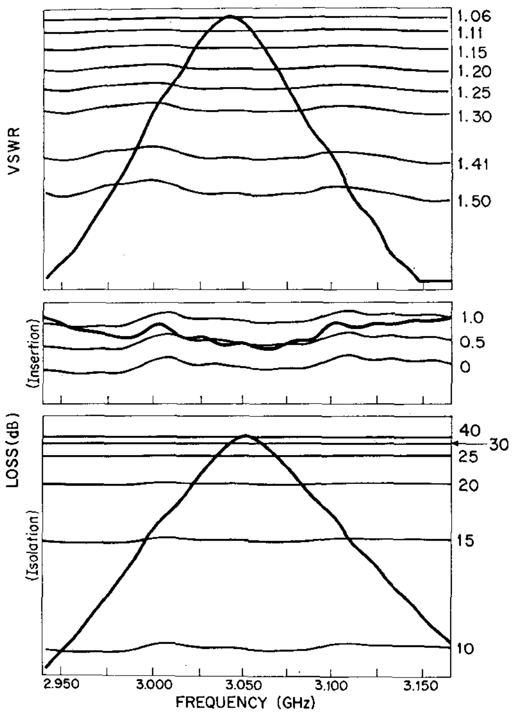

Fig. 7. Recorder tracings showing experimental ring circulator performance.

TABLE I

COMPARISON OF THEORY AND EXPERIMENT FOR THE RING CIRCULATOR

	Parameters	Theoretical	Experimental
Z_n	Shunt reactive impedance—ohms	50.0	47.0±1.0
arg ϵ	Average insertion phase factor—degrees	135.0	125.0
arg δ	Nonreciprocal phase factor—degrees	15.0	14.2
f_0	Center frequency of operation—GHz	3.000	3.045
Δf	Bandwidth for ≥20 dB isolation—MHz	87.0	60.0

here; for example, the magnitude and phase of the reflection E_1 from the input port conforms to prediction over a broad frequency range extending far beyond the nominal operating band.

V. Phase Shifter

The idea of combining a TEM transmission-stripline filter with ferrite elements to produce nonreciprocal phase shift may be regarded as an outgrowth of two familiar techniques: 1) field distortion, created by the use of dielectric inserts or other means, to produce circularly polarized components of the RF fields; and 2) the use of meander-line, helix, or other slow-wave structures to increase the amount of interaction with the ferrite. Such a combination has been used in traveling-wave masers[7] to make a resonance isolator.

A simple stripline "comb" filter is made by attaching a closely spaced series of stubs (teeth) along one edge of the stripline center conductor as illustrated in Fig. 8(a). The filter creates regions of circular polarization at points intermediate between adjacent teeth, thus giving rise to nonreciprocal phase shift when the ferrite is magnetized either parallel or perpendicular to the plane of the center conductor. A photograph of an experimental stripline comb filter is shown in Fig. 9. Typical curves giving experimental data on insertion[2] and differential phase[3] characteristics are presented in Fig. 10. Design principles for the lumped-element prototype of such a low-pass filter are presented in Guillemin[8] and elsewhere. Stripline phase shifters of this type have been designed which are compact,[4] have up to 35 degrees of nonreciprocal phase shift at 3.00 GHz as shown in

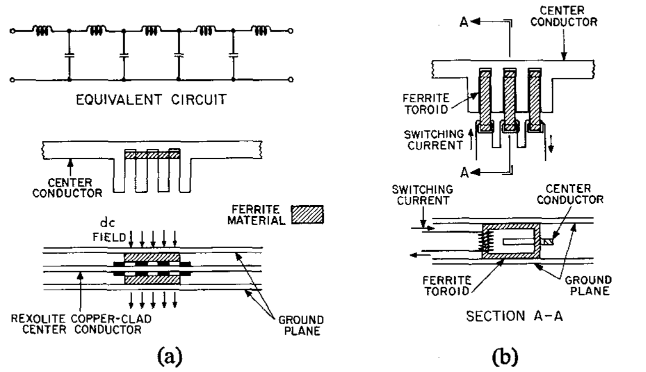

Fig. 8. Illustrating the design of a nonreciprocal stripline comb filter.

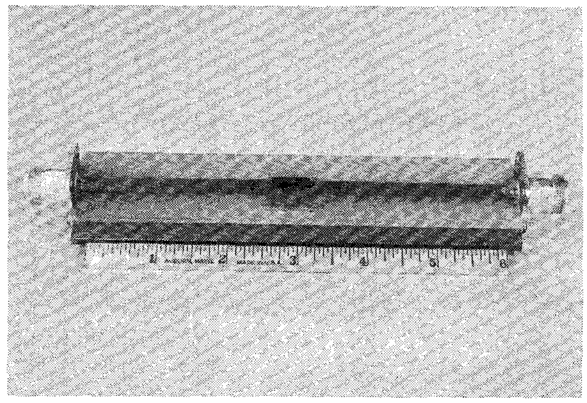

Fig. 9. An experimental stripline nonreciprocal comb filter with upper ground plane removed.

[2] The data of Fig. 10(a) differ from those referrred to in Table I because they do not include the electrical length of the line adjoining the filter in the ring structure.

[3] To fix the phase sign convention, we have adopted the view that the more natural choice of sign is positive for the time term $i\omega t$ and negative for position term $-i\beta z$. This means that the phase observed at a fixed point on the transmission line increases as time advances, while at a fixed time phase decreases, as we proceed away from the generator to portions of the wave which left the generator earlier. In computer work, however, it is convenient to suppress most of the negative signs by advancing all phases by appropriate multiples of 360 degrees. In the graphs of arg ϵ [Figs. 6(a), 7, 10(a), and 11(a)] the physically meaningful values are obtained by subtracting 360 degrees. The interpretation of the terms "increasing" and "decreasing" phase is as follows: the electrical length of a section of simple TEM line decreases (becomes more negative) with increasing frequency; in a backward-wave structure, the electrical length increases (becomes less negative) with increasing frequency.

[4] Length 0.615 inch, width 0.410 inch, ground plane spacing 0.312 inch, 1/16 inch rexolite copper-clad center conductor, number of sections, four.

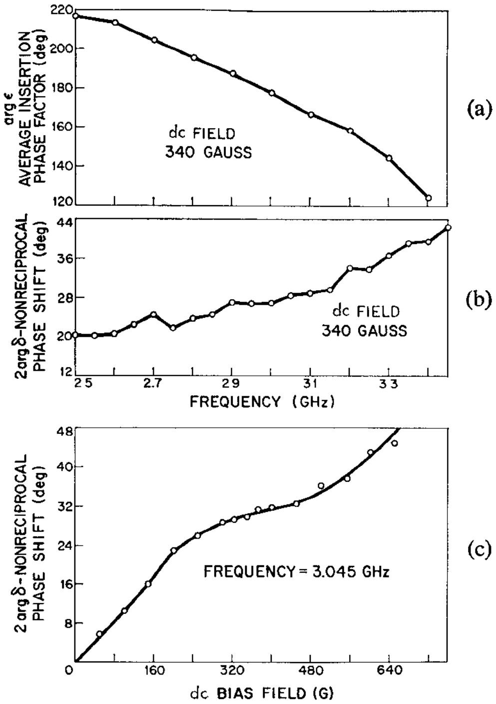

Fig. 10. Experimental data on a lowpass nonreciprocal stripline comb filter.

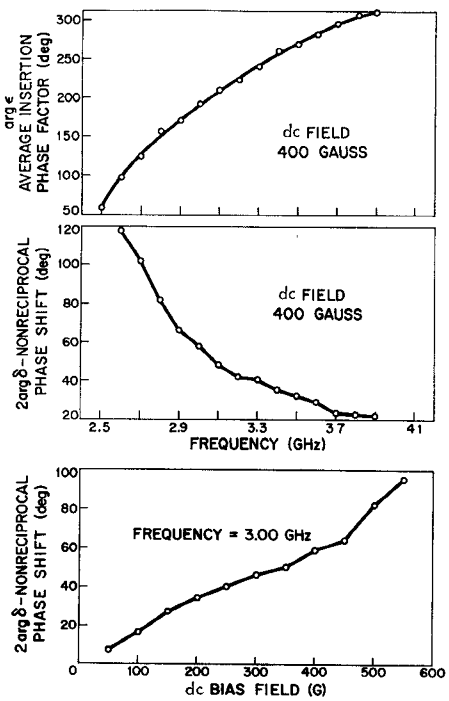

Fig. 11. Experimental data on a backward-wave nonreciprocal stripline comb filter.

Fig. 10(c), exhibit an insertion loss less than 0.3 dB and high efficiencies (up to 35 degrees with 0.014 cubic inch of ferrite —Trans-Tech G-1000). Because of the small volume of active material required, and the freedom available in the distribution of ferrite in relation to the transmission line, stripline comb filter nonreciprocal phase shifters have the added potential of permitting toroid configuration for high-speed switching [see Fig. 8(b)]. They also permit ferrite distributions which are favorable from the point of view of high peak and average power capability.

VI. Bandwidth

The performance of the experimental model can be predicted from the network theory by incorporating in the computation the appropriate theoretical (or experimental) frequency variation of the T-scattering coefficients and the actual δ and ϵ values from the experimental phase shifter. Insertion loss, isolation, and input match over the band of interest are determined by the overall circulator scattering coefficients E_1, E_2, and E_3. For a lossless circulator these parameters are, of course, not independent; in fact, as shown for example by Simon,[9] for low-loss circulators with good isolation we have $|E_3| \cong |E_1|$.

For the shunt capacitor T junction of Fig. 7(a), the simple reactive impedance variation $Z_n = 50 \times 3.0/f$ (f in GHz) of the shunt capacitors was assumed; arg δ and arg ϵ were obtained from actual phase-shifter measurements (Fig. 10). With the values of Z_n, arg δ, and arg ϵ as given in Table I holding at a band center of 3.00 GHz, the computed value of the 20 dB bandwidth was 87 MHz.

The narrow bandwidth of this prototype circulator is not inherent in the ring principle, but results from the failure of the dispersive characteristics of the phase shifters and of the T junctions to track properly. As can be seen from Figs. 7(a) and 10(a), when a T incorporating a shunt capacitance is used, the required variation of the insertion phase factor, arg ϵ, for circulation is increasing with increasing frequency. The curve of arg ϵ for the low-pass comb filter has the opposite slope; therefore, the proper relation between phases for circulation exists only over a very narrow band. By means of a proper design of the T junction, or the phase shifter, or both, the dispersions of these components can be made to agree in both sign and magnitude, thereby causing circulation to persist over a much broader band.

In accordance with this requirement, we investigated the possibility of using a high-pass "backward-wave" filter to produce the type of phase-shift dispersive characteristics necessary for broader band circulation. Experimental models were built for which arg ϵ increases with increasing frequency, as shown in Fig. 11. Preliminary observations show that such a phase shifter is quite satisfactory from the point of view of insertion loss and mechanical design, along with the capability of producing large amounts of nonreciprocal phase shift as shown in Fig. 11(c). Computations of circulator performance using these experimental data to characterize the phase shifters show that the bandwidth is increased as expected. These preliminary results indicate that there is considerable potential for further improvement in bandwidth and other specifications of the ring circulator.

VII. Comments on Ring Circulator Theories

The various previous proposals on ring circulators[1]–[3] were based on the intuitive expectation that circulation will result if the difference in phase shift along the two paths connecting input to output is an even multiple of 180 degrees, while the difference along the paths connecting the input to the isolated port is an odd multiple of 180 degrees. The rigorous scattering analysis performed in RNM[4] confirms this, but only as a special case of more general conditions under which perfect circulation can occur.

With regard to the possibility of obtaining perfect circulation even when the amount of nonreciprocal phase shift is small, reference has been made[1] to a theorem of Carlin[5] on realizability of nonreciprocal networks. The theorem does not assert, however, that circulation is impossible when the total nonreciprocal phase shift around the circumference of the ring is less than 180 degrees. Rather, it is an assertion regarding an equivalent circuit for the circulator in the special circumstance that all nonreciprocity is embodied in a single type of circuit element, namely, a *gyrator*, which, by its definition, introduces exactly 180 degrees of nonreciprocal phase. The theorem asserts that the minimum number of gyrators required in order to represent a three-port circulator by such a circuit is one. The logic of Carlin's method is illustrated in greater detail in another publication,[10] in which it is shown that the Carlin equivalent circuit of a two-port nonreciprocal phase shifter contains at least one gyrator, irrespective of the amount of nonreciprocity produced by the actual device. The conclusion is that the theorem does not exclude realizability of circulators with small nonreciprocal phase. The theoretical predictions of RNM, together with the experimental data on the device reported above, confirm that such devices are indeed realizable.[11]

VIII. Conclusions

The close agreement between the theoretical and experimental performance not only demonstrates that the network approach is a valid method and provides an efficient technique for circulator design, but confirms that zero and not 180 degrees is the lower bound for the total nonreciprocal phase necessary for perfect circulation. The rather narrow bandwidth of the present model is a direct result of the particular embodiment, namely, the shunt capacitive T junction and the "forward-wave" phase shifter. Bandwidth can be improved by either employing the "backward-wave" phase shifter or selecting other T configurations. The ring circulator concept suggests a means of achieving both high-speed switching and high microwave power-handling capability in a compact device. The comb-filter phase shifters provide an efficient means of obtaining nonreciprocal phase shift. In particular, these types of phase shifters offer possibilities for shaping both the insertion and differential phase characteristics through appropriate filter structure design.

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